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Annotated Chronological Bibliography

Submitted

[ESU]

Tiling space and slabs with acute tetrahedra, with David Eppstein and Alper Üngör.
We show it is possible to tile three-dimensional space using only tetrahedra with acute dihedral angles. We present several constructions to achieve this, including one in which all dihedral angles are less than 78 degrees, another which tiles a slab in space. Several of our examples come from tetrahedrally close-packed (TCP) crystal structures.

[S16]

Spherical Duals and Minkowski Sums.
We examine the Gauss map of a polyhedron, giving a spherical dual network. When this network is labeled with edge lengths, the original polyhedron can be recovered. Following a suggestion of Zongker and Hart, we show that the Minkowski sum of two polyhedra can be obtained simply by overlaying their labeled spherical duals.

[WKC+]

Pressures in Periodic Foams, with Denis Weaire, Norbert Kern, Simon J. Cox and Frank Morgan,
We show that pressures are periodic for any periodic foam, and that any planar foam with congruent bubbles is a (possibly sheared) hexagonal honeycomb with equal-pressure bubbles.

[FS] ***Visualizing a Sphere Eversion***, with George Francis.
The mathematical process of everting a sphere (turning it inside-out allowing self-intersections) is a grand challenge for visualization because of the complicated, ever changing internal structure. We have computed an optimal minimax eversion, requiring the least bending energy. Here we discuss techniques we used to help visualize this eversion for visitors to virtual environments and viewers of our video ‘‘The Optiverse’’ [SFL].

[GKS3]

Triunduloids: Embedded Constant Mean Curvature Surfaces with Three Ends and Genus Zero, with Karsten Große-Brauckmann and Rob Kusner. ArXiv eprint math.DG/0102183.
We classify complete, almost embedded surfaces of constant mean curvature, with three ends and genus zero (called *triunduloids*): they are classified by triples of points on the sphere whose distances are the asymptotic necksizes of the three ends. Since triunduloids are transcendental objects, and are not described by any ordinary differential equation, it is remarkable to have such a complete and explicit determination for their moduli space.

In Press

[S14]

The Tight Clasp. Electronic Geometry Model 2001.11.001, 2003, to appear.
This clasp is a numerical simulation of a tight (ropelength-minimizing) configuration of two linked arcs with endpoints in fixed parallel planes. Surprisingly, the arcs are not semicircles through each others’ centers.

[CKKS]

The Second Hull of a Knotted Curve, with Jason Catarella, Greg Kuperberg and Rob Kusner. *Amer. J. Math.*, 2003, to appear. ArXiv eprint math.GT/0204106.
We define the second hull of a space curve, consisting of those points which are doubly enclosed by the curve in a certain

sense. We prove that any knotted curve has nonempty second hull. We relate this to recent results on thick knots, quadrisecants, and minimal surfaces.

[GS]

Cubic Polyhedra, with Chaim Goodman-Strauss. In *Discrete Geometry* (Kuperberg festschrift), Marcel Dekker, 2003, to appear. ArXiv eprint math.DG/0205145.

A cubic polyhedron is a polyhedral surface whose edges are exactly all the edges of the cubic lattice. Every such polyhedron is a discrete minimal surface, and it appears that many (but not all) of them can be relaxed to smooth minimal surfaces (under an appropriate smoothing flow, keeping their symmetries). Here we give a complete classification of the cubic polyhedra. Among these are five new infinite uniform polyhedra and an uncountable collection of new infinite semi-regular polyhedra. We also consider the somewhat larger class of all discrete minimal surfaces in the cubic lattice.

[S15]

The Aesthetic Value of Optimal Geometry. In *The Visual Mind II*, MIT Press, 2003, to appear.

Geometric optimization problems arise physically in many situations: material interfaces, for instance, usually minimize some surface energy. Curves and surfaces which are optimal for geometric energies often have aesthetically pleasing shapes. Computer simulation of such optimal geometry can be useful for mathematicians seeking insight into the behavior of minimizers, for designers looking for graceful shapes and attractive graphics, and for scientists modeling nature.

[FLS2]

Making the Optiverse: A Mathematician's Guide to AVN, a Real-Time Interactive Computer Animator, with George Francis and Stuart Levy. In *Mathematics, Art, Technology, Cinema*, Springer, 2003, to appear.

Our 1998 video ‘‘The Optiverse’’ [SFL] illustrates an optimal sphere eversion, computed automatically by minimizing an elastic bending energy for surfaces. This paper describes AVN, the custom software program we wrote to explore the computed eversion. Various special features allowed us to use AVN also to produce our video: it controlled the camera path throughout and even rendered most of the frames.

Published

[FGKSS]

ALICE on the Eightfold Way: Exploring Curved Spaces in an Enclosed Virtual Reality Theater, with George Francis, Camille Goudeseune, Hank Kaczmarski and Ben Schaeffer. In *Visualization and Mathematics III*, Springer, 2003, pp 307-317 and 431.

We describe a collaboration between mathematicians interested in visualizing curved three-dimensional spaces and researchers building next-generation virtual-reality environments such as ALICE, a six-sided, rigid-walled virtual-reality chamber. This environment integrates active-stereo imaging, wireless position-tracking and wireless-headphone sound. To reduce cost, the display is driven by a cluster of commodity computers instead of a traditional graphics supercomputer. The mathematical application tested in this environment is an implementation of Thurston's eight-fold way; these eight three-dimensional geometries are conjectured to suffice for describing all possible three-dimensional manifolds or universes.

[CKS2]

On the Minimum Ropelength of Knots and Links, with Jason Catarella and Rob Kusner. *Inventiones Math.* **150**:2, 2002, pp 257-286. (Published online as DOI 10.1007/s00222-002-0234-y.) ArXiv eprint math.GT/0103224.

The ropelength of a knot is the quotient of its length and its thickness, the radius of the largest embedded normal tube around the knot. We prove existence and regularity for ropelength minimizers in any knot or link type; these are $C^{1,1}$ curves, but need not be smoother. We improve the lower bound for the ropelength of a nontrivial knot, and establish new ropelength bounds for small knots and links, including some which are sharp.

[EGSU]

Building space-time meshes over arbitrary spatial domains, with Jeff Erickson and Damrong Guoy and Alper Üngör. In *Proceedings of the 11th International Meshing Roundtable*, Sandia, 2002, pp 391-402. ArXiv eprint cs.CG/0206002.

We present an algorithm to construct meshes suitable for space-time discontinuous Galerkin finite-element methods. Our method generalizes and improves the "Tent Pitcher" algorithm of Üngör and Sheffer. Given an arbitrary simplicially meshed spatial domain of any dimension and a time interval, our algorithm builds a simplicial mesh of their space-time product domain, in constant time per element. Our algorithm avoids the limitations of previous methods by carefully adapting the durations of space-time elements to the local quality and feature size of the underlying space mesh.

[S13]

Sphere Eversions: from Smale through "The Optiverse". In *Mathematics and Art: Mathematical Visualization in Art and Education*, (Maubeuge 2000), Springer, 2002, pp 201-212 and 311-313.

This is a revised and updated version of [S8].

[S12]

Approximating Ropelength by Energy Functions. In *Physical Knots* (Las Vegas 2001), AMS Contemp. Math., 2002, 181--186. ArXiv eprint math.GT/0203205.

The ropelength of a knot is the quotient of its length by its thickness. We consider a family of energy functions for knots, depending on a power p, which approach ropelength as p increases. We describe a numerically computed trefoil knot which seems to be a local minimum for ropelength; there are nearby critical points for the p-energies, which are evidently local minima for large enough p.

[FLS1]

The Optiverse: una guida ai matematici per AVN, programma interattivo di animazione, with George Francis and Stuart Levy. In *Matematica, arte, tecnologia, cinema*, Springer, 2002, pp 37-51.

An Italian translation of [FLS2].

[S11]

Rescalable Real-Time Interactive Computer Animations. In *Multimedia Tools for Communicating Mathematics*, Springer, 2002, pp 311-314.

Animations are one of the best tools for communicating three-dimensional geometry, especially when it changes in time through a homotopy. For special-purpose animations, custom software is often necessary to achieve real-time performance. This paper describes how, in recent years, computer hardware has improved, and libraries have been standardized, to the point where such custom software can be easily ported across all common platforms, and the performance previously found only on high-end graphics workstations is available even on laptops.

[CDES2]

Dynamic Skin Triangulation, with Ho-Lun Cheng, Tamal K. Dey, and Herbert Edelsbrunner, *Discrete and Computational Geometry* **25**, 2001, pp 525-568. CMP 1 838 419

This paper describes an algorithm for maintaining an approximating triangulation of a deforming smooth surface in space. The surface is the envelope of an infinite family of spheres defined and controlled by a finite collection of weighted points. The triangulation adapts dynamically to changing shape, curvature, and topology of the surface.

[CDES1]

Dynamic Skin Triangulation, with Ho-Lun Cheng, Tamal K. Dey, and Herbert Edelsbrunner, *Proc. 12th Ann. ACM-SIAM Sympos. Discrete Alg.*, 2001 Jan, pp 47-56.

This is the 10-page announcement of [CDES2].

[GKS2]

Constant Mean Curvature Surfaces with Three Ends, with Karsten Große-Brauckmann and Rob Kusner, *Proc. Natl. Acad. Sci.* **97**:26, 2000 Dec 19, pp 14067-14068. ArXiv eprint

math.DG/9903101. MR 2001j:53009
We announce the classification of triunduloids given in [GKS3].

[FTY+]
Tomographic Imaging of Foam, with Fetterman, Tan, Ying, Stack, Marks, Feller, Cull, Munson, Thoroddsen and Brady, *Optics Express* **7**:5, 2000 Aug 28, pp 186-197.
We explore the use of visual-light tomography to create three-dimensional volume images of small samples of soap foams. We place the foam sample on a rotating stage, and acquire a sequence of images. The tomographic algorithm corrects for the distortion of the curved plexiglass container. Such reconstructions allow comparison of physical foam experiments with computer simulations of foam diffusion in the Surface Evolver.

[AST]
Foam Evolution: Experiments and Simulations, with Hassan Aref and Sigurdur T. Thoroddsen, in *Proc. NASA 5th Microgravity Fluid Physics Conf.*, Aug 2000, pp 99-100.
This extended abstract describes relations between experimental observations, mathematical models, and numerical simulations of foams, including the dynamics of reconnection events, phase transitions in compressible foams, tomographic reconstruction of foams, and the combinatorics of TCP foams.

[S10]
New Tetrahedrally Close-Packed Structures, in *Proc. Eurofoam 2000 (Delft)*, June 2000, pp 111-119.
This article in the proceedings of the Third Euroconference on Foams describes a new construction for TCP structures and their associated foams. This construction allows the creation of TCP triangulations of different 3-manifolds, which are convex combinations of the known basic TCP structures exactly when the manifold is flat. This is a first step in understanding the relation between combinatorics and topology for three-manifolds. The class of TCP triangulations is of further interest because most can be made with all dihedral angles acute. This property is important for many meshing applications, for good numerical analysis, but methods of constructing acute triangulations were previously unknown.

[S9] ***Foams and Bubbles: Geometry and Simulation***, *Intl. J. Shape Modeling*, **5**:1, 1999, pp 101-114.
This invited contribution to a special issue edited by Michele Emmer is adapted and updated from [S7].

[S8] ***"The Optiverse" and Other Sphere Eversions***, in *ISAMA 99*, Univ. Basque Country, 1999, pp 471-479. Reprinted in *Bridges 1999*, Southwestern Coll., Kansas, 1999, pp 265-274. Full-color version published in the online journal *Visual Mathematics*, **1**:3, September 1999. ArXiv e-print math.GT/9905020. Also available in an HTML version.
For decades, the sphere eversion has been a classic subject for mathematical visualization. Our 1998 video "The Optiverse" [SFL] shows geometrically optimal eversions created by minimizing elastic bending energy. This paper contrasts these minimax eversions with earlier ones, including those by Morin, Phillips, Max, and Thurston. Our minimax eversions were automatically generated by flowing downhill in energy using Brakke's Evolver.

[CKS1]
Crossing Numbers of Tight Knots, with Jason Cantarella and Rob Kusner, *Nature* **392**:6673, 1998, pp 237-238.
This note shows that, contrary to a conjecture published by some biophysicists a year earlier in *Nature*, there is not a linear relation between the minimum crossing number of a knot and its minimal ropelength. Instead, we construct examples where the crossing number grows like the 4/3 power of ropelength, the optimum possible.

[SFL]
The Optiverse, with George Francis and Stuart Levy, in *VideoMath Festival at ICM'98*, Springer, 1998, 7-minute video.
This video shows the minimax sphere eversions described in [FSK+] and [FSH]. These are geometrically optimal ways to turn a sphere inside out, computed by minimizing Willmore's elastic bending energy for surfaces. The video was chosen for the exclusive Electronic Theater at SIGGRAPH 98, and was selected by the jury for presentation at ICM'98. It has

been the subject of an article in *Science* and others in magazines on three continents.

[S7] ***The Geometry of Bubbles and Foams***, in *Foams and Emulsions* (NATO ASI volume E 354), Kluwer, 1999, pp 379-402. MR 2000b:53015
This survey records my invited series of lectures at an interdisciplinary NATO school on foams (Cargèse, 1996) organized by J.F. Sadoc and N. Rivier. It reviews the theory of constant-mean-curvature surfaces, the combinatorics of foams and their dual triangulations, their relation to crystal structures, and the current status of the Kelvin problem and related results.

[S6] ***Knot Energies***, in *VideoMath Festival at ICM'98*, Springer, 1998, 3-minute video.
This video, selected by an international jury to be shown at ICM'98, shows examples of Möbius-energy minimization for knots and links, as described in [KS1].

[OS]
The beta-Sn Dual Structure: A 4-Connected Net Based on a Packing of Simple Polyhedra with 18 Faces, with Michael O'Keeffe, *Z. Kristallographie* **213**, 1998, pp 374-376.
This crystallography paper describes a new three-dimensional structure arising out of discussions on foam structures and their relations to crystals.

[KS4]
On the Distortion and Thickness of Knots, with Rob Kusner, in *Topology and Geometry in Polymer Science* (IMA volume 103), Springer, 1998, pp 67-78. ArXiv eprint dg-ga/9702001. MR 99i:57019
We formulate and compare different rigorous definitions for the thickness of a space curve, that is, the diameter of the thickest tube that can be embedded around the curve. One definition involves Gromov's notion of the distortion of the embedding of the curve. Our definitions are especially useful because they are non-zero for polygonal curves, and thus are easier to measure in computer simulations of knots minimizing their ropelength (length divided by thickness).

[KS3]
Möbius-invariant Knot Energies, with Rob Kusner, in *Ideal Knots* (MR 2000j:57018), World Scientific, 1998, pp 315-352. CMP 1 702 037
This is an updated reprinting of [KS1], as an invited contribution to a volume in the "Series on Knots and Everything", edited by Stasiak, Katritch, and Kauffman.

[FSH]
Computing Sphere Eversions, with George Francis and Chris Hartman, in *Mathematical Visualization* (MR 99k:65005), Springer, 1998, pp 237-255. CMP 1 677 675
This paper describes how to adapt the methods of [FSK+] to compute the minimax sphere eversions of higher-order symmetry which are also shown in "The Optiverse" [SFL]. In particular, we must use symmetry features of the evolver [BS] to perform the computations.

[GKS1]
Constant Mean Curvature Surfaces with Cylindrical Ends, with Karsten Große-Brauckmann and Rob Kusner, in *Mathematical Visualization* (MR 99k:65005), Springer, 1998, pp 107-116.
CMP 1 677 699
Almost embedded CMC surfaces have ends asymptotic to Delaunay unduloids; therefore they have finite total absolute curvature if and only if all of their ends are asymptotic to cylinders. A conjecture due to Rick Schoen had been that the cylinder should be the only such surface, but here we give good numerical evidence against that conjecture. By gluing together truncated triunduloids, we construct surfaces with, say, thirty ends, all cylindrical.

[KS2]
Comparing the Weaire-Phelan Equal-Volume Foam to Kelvin's Foam, with Rob Kusner, *Forma* **11**:3, 1996, pp 233-242. Reprinted in *The Kelvin Problem*, Taylor & Francis, 1996, pp 71-80.
MR 99e:52031

Lord Kelvin conjectured a foam structure as the optimal partition of space into equal-volume cells, with least surface area. A century later, Weaire and Phelan discovered an equal-volume foam which numerically seemed better than Kelvin's candidate. Our contribution to this special volume edited by Denis Weaire shows how to rigorously prove that the Weaire-Phelan foam does beat Kelvin's foam.

[MS]

In Memoriam Frederick J. Almgren Jr., 1933-1997: On Being a Student of Almgren's, with Frank Morgan, *Experimental Math.* **6**:1, 1997, pp 8-10. CMP 1 464 578

These descriptions of what is was like to be Almgren's student, published alongside reminiscences by mathematicians who knew him in other ways, show the evolution of Almgren's work over the course of a decade, as he grew to appreciate the value of computers in solving geometric problems in pure mathematics.

[FSK+]

The Minimax Sphere Eversion, with George Francis, Rob Kusner *et al*, in *Visualization and Mathematics* (MR 99g:68212), Springer, 1997, pp 3-20. CMP 1 607 221

Here we explain the mathematical theory behind the geometrically optimal minimax sphere eversion shown in "The Optiverse" [SFL]. This eversion is accomplished by numerically modeling the gradient flow for the Willmore energy, starting from the lowest saddle point and flowing down to the round sphere.

[BS]

Using Symmetry Features of the Surface Evolver to Study Foams, with Ken Brakke, in *Visualization and Mathematics* (MR 99g:68212), Springer, 1997, pp 95-117. CMP 1 607 360

This report describes how certain new features we have added to the Surface Evolver can be used to take advantage of symmetries of a surface being modeled. As a test case, we describe how to accurately model the Kelvin foam and the Weaire-Phelan foam, which is a better partition of space into equal-volume cells (see [KS2]).

[SM]

Open Problems in Soap-Bubble Geometry, editor, with Frank Morgan, *International J. Math.* **7**:6, 1996, pp 833-842. MR 98a:53014

This list collects and organizes a long list of open problems posed by participants at a special session on Soap-Bubble Geometry at the AMS MathFest in Burlington in 1994, as well as further problems suggested by the editors.

[KS1]

Möbius Energies for Knots and Links, Surfaces and Submanifolds, with Rob Kusner, in *Geometric Topology*, International Press, 1996, pp 570-604. MR 98d:57014

In this paper, we give a nicer explanation of the Möbius-invariance of the knot energy studied by Freedman, He and Wang, and extend it to higher-dimensional submanifolds. We also give the first examples of knot and link types with several distinct critical points for this energy. We include a table and illustrations of numerically computed energy-minimizing configurations of all knots and links through eight crossings.

[CGLS]

Elliptic and Parabolic Methods in Geometry, editor, with Ben Chow, Bob Gulliver and Silvio Levy. Published by AK Peters, 1996. MR 97f:58004

This book is the proceedings volume from a five-day workshop we organized, held in Minneapolis in 1994. Twelve contributions by outstanding geometers convey the potential of using computers in studying a wide range of open questions in geometry. Topics include curvature flows, harmonic maps, liquid crystals, and CMC surfaces.

[S5] ***Sphere Packings Give an Explicit Bound for the Besicovitch Covering Theorem***, *J. Geometric Analysis* **4**:2, 1994, pp 219-231. MR 95e:52038

This paper, which arose from a lemma used in my dissertation, examines a standard proof of the Besicovitch Covering Theorem from the point of view of finding the optimal constant, which turns out to also be the answer to a sphere-packing problem: how many unit spheres fit into a ball of radius five? In high dimensions, I review the best asymptotic bounds known. In two dimensions, I show the answer is 19, while in three dimensions, I give the best upper and lower bounds known.

[MSL]

Monotonicity Theorems for Two-Phase Solids, with Frank Morgan, Francis Larché, *Arch. Rat. Mech. Anal.* **124**:4, 1994, pp 329-353. MR 94m:73072

Here we give a rigorous mathematical proof of some observations about metal alloy systems at concentrations for which two phases coexist. If there were no cost involved in mixing the phases, each phase would be at a fixed concentration, even as the overall concentration *c* of the two metals in the alloy varied. Here we explain, using techniques of convex analysis, the counterintuitive fact that, with a mixing cost, the individual concentrations vary inversely with *c*. Along the way, we find several interesting lemmas about minima of functions of several variables and parameters.

[S4] ***Computing Hypersurfaces Which Minimize Surface Energy Plus Bulk Energy***, in *Motion by Mean Curvature and Related Topics*, de Gruyter, 1994, pp 186-197. MR 95h:49072

My dissertation proved an approximation theorem for area-minimizing hypersurfaces in the context of geometric measure theory. This kind of approximation is especially useful to prove the feasibility of algorithms to find area-minimizing surfaces without *a priori* knowing their topology. This paper (appearing in the proceedings of a 1992 conference in Trento) shows that the approximation theorem and algorithms can be extended to the case where the minimization involves not just a surface energy, but also bulk terms like volume or gravity.

[HKS]

Minimizing the Squared Mean Curvature Integral for Surfaces in Space Forms, with Lucas Hsu and Rob Kusner, *Experimental Math.* **1**:3, 1992, pp 191-207. MR 94a:53015

We report on the results of the first computer simulations of Willmore surfaces, using Brakke's Evolver. The numerical evidence supports Willmore's conjecture about the minimizing torus, and suggests that certain Lawson surfaces minimize for higher genus. These simulations have been of interest to biophysicists studying lipid vesicles.

[S3] ***Using Max-Flow/Min-Cut to Find Area-Minimizing Surfaces***, in *Computational Crystal Growers Workshop*, AMS Sel. Lect. Math., 1992, pp 107-110 plus video.

This video uses algorithm animation to illustrate how the algorithm described in my dissertation uses max-flow/min-cut techniques to find approximations to area-minimizing surfaces, without knowing their topology in advance; it appears in the proceedings of a conference organized by Jean Taylor.

[AS]

Visualization of soap bubble geometries, with Fred Almgren, *Leonardo* **24**:3/4, 1992, pp 267-271. Reprinted in *The Visual Mind* (MR 94h:00013), MIT Press, 1993, pp 79-83. CMP 1 255 841

This paper, in a special volume edited by Michele Emmer, surveys results on the geometry of bubble clusters, and describes my rendering algorithm for photorealistic computer graphics of soap film, also used later in "The Optiverse" [SFL].

[S2] ***Crystalline Approximation: Computing Minimum Surfaces via Maximum Flows***, in *Computing Optimal Geometries*, AMS Selected Lectures in Math., 1991, pp 59-62 plus video.

This video shows, using a two-dimensional example, how the approximation theorem proved in my dissertation works to find approximately area-minimizing surfaces; it appears in the proceedings of an AMS special session organized by Almgren and Taylor.

[S1] ***Generating and Rendering Four-Dimensional Polytopes***, *The Mathematica Journal* **1**:3, 1991, pp 76-85.

This expository paper shows a nice way to generate coordinates for the regular polytopes in three and four dimensions, and describes how to picture the four-dimensional polytopes via stereographic projection as bubble clusters in three-space. It is illustrated with computer graphics using the algorithm described in [AS].

[ABST]

Computing Soap Films and Crystals, with Fred Almgren, Ken Brakke, Jean Taylor, in *Computing Optimal Geometries*, AMS Selected Lectures in Math., 1991, 14-minute video.

This video, which we produced at the Geometry Supercomputer Center while I was in graduate school, shows some early computations with Brakke's evolver, computations done with my three-dimensional Voronoi cell code, and crystalline minimal surfaces computed by Taylor.

[ST]

Animating the Heat Equation: A Case Study in Mathematica Optimization, with Matt Thomas, *The Mathematica Journal* **1**:1, 1990, pp 80-84.

When I was asked to referee a submission by Thomas to the first issue of *The Mathematica Journal*, I found I could optimize his code, resulting in almost a thousand-fold speedup. The main thrust of the published joint article became a description of these optimization techniques.

[LMS]

Some Results on the Phase Behavior in Coherent Equilibria, with Francis Larché, Frank Morgan, *Scripta Metallurgica* **24**:3, 1990, pp 491-493.

In some metal alloy systems two phases coexist for certain concentrations. This metallurgy paper explains the counterintuitive fact that the concentration in each phase varies inversely with the overall concentration. The mathematical details are given in [MSL].

[S0] ***A Crystalline Approximation Theorem for Hypersurfaces***, Princeton University *Ph.D.* thesis, 1990; Geometry Center report GCG 22.

My dissertation shows that any hypersurface can be approximated arbitrarily well by polygons chosen from the finite set of facets of an appropriate cell complex, with restricted orientation and positions. Thus we can approximate the problem of finding the least-area surface on a given boundary by a finite network-flow problem in linear programming. This gives an effective algorithm for finding such surfaces, without knowing their topology in advance. Pieces of my dissertation, and related results, appear in [S2], [S3], [S4], [S5], but the main section of the work has not yet been published elsewhere.