Consider the following questions:

• Compute the partial derivatives of \( f \).
• Compute the distortion in the horizontal direction.
• Compute the distortion in the vertical direction.
• Compute the distortion in the direction \((1,1)\).
• Is the distortion the same in every direction at a given point?
• Are the partial derivatives of \( f \) perpendicular at each point?
• Is \( f \) conformal?
• Is it a local isometry?
• Is \( f \) one-to-one?
• Is \( f \) onto?
• Does \( f \) take lines to lines?
• Describe in words what the map does.

(1) Answer these questions for the function \( f \) from \((-\pi,\pi) \times (-1,1) \subset \mathbb{E}^2 \) to \( S^2 \) defined as follows (we are using \((u,v)\) as coordinates on \( \mathbb{E}^2 \) rather than the usual \((x,y)\)):

\[
F(u,v) := (\sqrt{1-v^2} \cdot \cos u, \sqrt{1-v^2} \cdot \sin u, v).
\]

(2) Let \( g : \mathbb{R} \to \mathbb{R} \) be any differentiable function and let \( G(x,y) = (x,g(y)) \). Answer the questions for the new function

\[
H(x,y) = F \circ G(x,y) = (\sqrt{1-g(y)^2} \cdot \cos x, \sqrt{1-g(y)^2} \cdot \sin x, g(y)).
\]

Use the chain rule:

\[
H_x = (F \circ G)_x = F_u u_x + F_v v_x,
\]

\[
H_y = (F \circ G)_y = F_u u_y + F_v v_y,
\]

remembering that \((u,v) = G(x,y) = (x,g(y))\).

Your answer will, of course, depend on \( g \).