Math 302

Homework #9, Fall 2001

Due Wednesday November 14, at the beginning of class.

(1) For each of the following, decide whether or not such a polygon can exist. If not, explain why not. If so, show that it is possible (a clear drawing, properly labelled, is sufficient, or you can explain in words). The program NonEuclid from the computer lab may be helpful.

(a) A rectangle in $\mathbb{H}^2$. (We define a rectangle as a quadrilateral with a right angle at each vertex.)
(b) A rectangle in $\mathbb{S}^2$.
(c) A pentagon (five-sided polygon) in $\mathbb{H}^2$ having a right angle at every vertex.
(d) A triangle of area $2\pi \rho^2$ in $\mathbb{H}^2$.
(e) A triangle with three right angles in $\mathbb{S}^2$.

(2) Derive a formula for the angle sum of a simple quadrilateral in $\mathbb{S}^2$ in terms of the area of the quadrilateral.

(3) In the upper half plane $U$, consider the ideal triangle whose sides are the semicircle $x^2 + y^2 = 4$, the ray $x = 2$ and the ray $x = -2$. Sketch the image of this ideal triangle under inversion in the semicircle $x^2 + y^2 = 1$ and explain how you arrived at your sketch. What is the (hyperbolic) area of the original ideal triangle? What is the (hyperbolic) area of the ideal triangle you get after inversion? How are these two areas related and why?