Foam Evolution: Experiments and Simulations

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We describe several areas of work on the fluid physics of foam evolution and flow, including experimental observations, mathematical models, and numerical simulations. We have made detailed observations of the topological reconnections that occur in an evolving foam, and propose a new model for the flow there. We describe the phase transition which can happen as a foam is expanded until it exerts negative pressure on its container. We are exploring new optical tomography techniques to observe an evolving foam and reconstruct the three-dimensional structure. We compare the evolution of a real foam with simulations in Brakke's Evolver. We can also simulate infinite periodic foams, including new mathematical possibilities for TCP structures.

RECONNECTIONS
The Plateau rules say that in any froth or foam, bubbles meet in threes along the so-called Plateau borders; these curves then meet in fours at the nodes. As a foam evolves (mainly due to diffusion of gas between the bubbles) the nodes will sometimes meet, resulting in a combined node not obeying the Plateau rules, which thus immediately breaks apart into several nodes, connected by a new film or Plateau border which expands very rapidly.

Using high-speed video equipment, we have made accurate observations of these reconnection events, which happen over time scales less than a tenth of a second. We observed reconnections in coarsening foams, but also in a special adjustable triangular-prism wire frame which allows control of a single reconnection. We note that the wetness (or liquid fraction) can have a great effect on the speed and sequence of reconnections, since it changes the size of the Plateau borders and nodes.

We have used our experimental data to construct a simple dynamical model for a reconnection event, incorporating Marangoni effects which are due to the rapid stretching of the surface and the resultant change in surfactant concentration.

In a coarsening forth, the slow diffusion of gas between adjacent bubbles of differing pressure is punctuated by these rapid reconnections. Because each reconnection makes a large local change in geometry, it is likely to lead immediately to further reconnections. Thus reconnections occur in avalanches. We have recorded, with multiple video cameras, the evolution of a coarsening foam over a period of days, and scanned the tapes to collect statistics about these avalanches.

PHASE TRANSITIONS
The structure of a foam results from a balance between pressure in the gas bubbles and surface tension in the liquid films. Under normal conditions, a foam in a box exerts an outward force on the walls of the box, due to the pressure. But when surface tension is much more important (for instance if the foam is greatly expanded from its original volume), the net force on the walls will be inward. This means that the ambient pressure of the foam should be considered negative.
In this regime of negative pressure, an interesting phase transition can happen. The bubbles segregate into two phases, one consisting of a few huge bubbles, and the other consisting of many small bubbles. We have developed a theory to explain this phase transition, which agrees very well with simulations made in the Evolver. We hope to find an appropriate physical system with which we can experiment with this transition.

In many real-world applications of foams, it is very important to control the size of the foam cells: large voids typically make a foamed material much less useful. It seems likely that the undesirable large cells sometimes found in real foams are the result of the phase transition we have described. A better understanding will hopefully lead to processing techniques which can avoid this transition.

**TOMOGRAPHY**

In collaboration with David Brady of the Beckman Institute’s imaging group, We have started an project to use visible-light tomography to capture the three-dimensional structure of a foam. Preliminary results are promising: under appropriate lighting conditions the tomographic Radon transform results in a volumetric density profile from which Plateau border positions can be extracted.

We have already written and tested software which will reconstruct the entire foam structure (supplying information about the faces and cells) from the Plateau borders (or perhaps just the node positions) alone. This uses the known geometry of foams (with equal angle conditions guaranteed by the Plateau rules) to find successively the edges, faces, and cells, and outputs these in Evolver format.

The tomographic process will allow us to automatically track the coarsening of a foam over a period of many hours, and compare this with a simulation of diffusive coarsening in the Evolver. There are many interesting questions about the statistics of cell growth in three dimensions; we will be able to address these.

**SIMULATIONS AND TCP STRUCTURES**

We have recently discovered new mathematical constructions for foam structures in the class known as tetrahedrally close-packed (TCP). These are interesting in that the least-energy (ground state) configuration for a foam with equal-volume cells is evidently one of the simplest TCP structures (though this remains unproven mathematically).

Our new structures form an interesting intermediate class, inbetween random foams and completely ordered foams (like Kelvin’s foam). We can model them in the Evolver, and use them as interesting starting points for diffusion simulations.